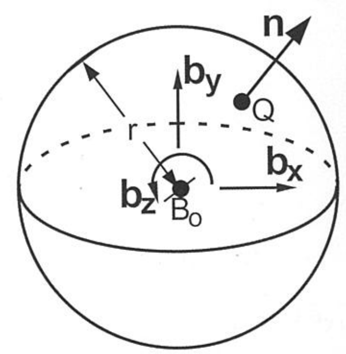
	Ellipse: $a = 2$ and $b = 4$	Circle: $a = 2$ and $b = 2$
Result:	$\vec{n} = 1.0\,\hat{b}_x + 0.433\,\hat{b}_y$	$\vec{n} = 0.5\,\hat{b}_x + 0.866\,\hat{b}_y$
	$\vec{t} = -0.433\,\hat{b}_x + 1.0\,\hat{b}_y$	$\vec{t} = -0.866\,\hat{b}_x + 0.5\,\hat{b}_y$

When $a = b$ (the ellipse is a circle), $\vec{n}$ is always parallel to $\vec{r}^{\,Q/B_o}$ **True/False**
When $a \neq b$ (the ellipse is not a circle), $\vec{n}$ is always parallel to $\vec{r}^{\,Q/B_o}$ **True/False**

(h) **Optional****: Show how the definition of an ***ellipse*** results in $F(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$.

3.14 Optional**: Normal to a sphere

A ***sphere*** can be defined as the locus of points that are a distance r (called the ***sphere's radius***) from a point B_o (called the ***sphere's center***). For example, the figure to the right shows a sphere of radius r that is centered at point B_o.



The position of a point Q on the sphere's periphery from point B_o can be expressed in terms of the scalars x and y as

$$\vec{r}^{\,Q/B_o} = x\,\hat{b}_x + y\,\hat{b}_y + z\,\hat{b}_z$$

where $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ are right-handed, orthogonal, unit vectors.
Show how a sphere's definition results in the following relationship.
Result: $F(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0$

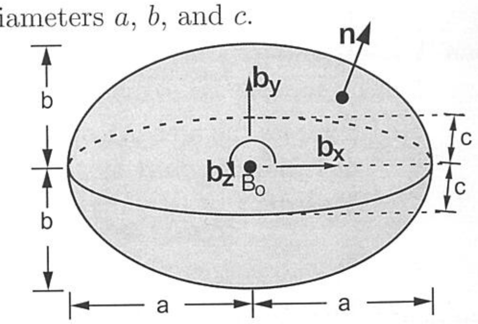
When a scalar function F describes the boundary of an object, the spatial gradient $\vec{\nabla} F$ is normal to the boundary. With $\vec{r}^{\,Q/B_o} = x\,\hat{b}_x + y\,\hat{b}_y + z\,\hat{b}_y$, $\vec{\nabla} F$ can be expressed as

$$\vec{\nabla} F \underset{(6.12)}{=} \frac{\partial F}{\partial x}\hat{b}_x + \frac{\partial F}{\partial y}\hat{b}_y + \frac{\partial F}{\partial z}\hat{b}_z$$

Use $\vec{\nabla} F$ to calculate an outward normal vector $\vec{n}$ at point Q in terms of x, y, r, etc.
Result: $\vec{n} = \quad \hat{b}_x + \quad \hat{b}_y + \quad \hat{b}_y$

3.15 Optional**: Normal to an ellipsoid

The following figure shows a point Q on a ellipsoid of semi-diameters a, b, and c.



Right-handed, orthogonal, unit vectors $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ are directed with $\hat{b}_x$ pointing right along the ellipsoid's major axis and $\hat{b}_y$ pointing up along the ellipsoid's minor axis.

The position of Q from B_o (the ellipsoid's center) can be expressed in terms of the scalars x, y, and z as

$$\vec{r}^{\,Q/B_o} = x\,\hat{b}_x + y\,\hat{b}_y + z\,\hat{b}_z$$

Calculate x, y, z when $x = \frac{a}{2}$, $z = \frac{c}{2}$, $a = 3$, and $b = c = 2$.
Result: $x = 1.5 \quad y = 2\sqrt{\frac{1}{2}} \approx 1.414214 \quad z = 1$

Determine an outward normal vector $\vec{n}$ at point Q in terms of x, y, z, a, b, c.
Calculate the unit vector in the $\vec{n}$ direction when $x = \frac{a}{2}$, $z = \frac{c}{2}$, $a = 3$, and $c = 2$.

	General case	Unit vector with numerical values
Result:	$\vec{n} = \quad \hat{b}_x + \quad \hat{b}_y + \quad \hat{b}_z$	$0.359\,\hat{b}_x + 0.762\,\hat{b}_y + 0.539\,\hat{b}_z$

4.1 Circles, π, degrees, radians, arc-length. (Section 1.4).

- **Draw** a circle with radius r and calculate its circumference in terms of r.
- Using the circle, define the irrational number π. Approximate π in radians and degrees.
- **Draw** a 45° circular arc with radius r and calculate its arc-length s in terms of r.

Draw a circle of radius r — Draw a 45° circular arc

$\pi \triangleq$

$\pi \approx$

$\pi = \quad °$

Circumference =

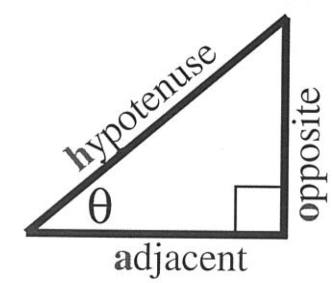
Arc-length: $s =$

Provide an explanation as to why there are 360° in a circle.
Explain:

4.2 SohCahToa: Sine, cosine, tangent as ratios of sides of a right triangle. (Section 1.5).

The following shows a ***right triangle*** (a triangle with a ***right angle***, i.e., a 90° angle) with one of its angles labeled as θ. Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle's longest side (opposite the 90° angle).
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ



$\sin(\theta) \triangleq$ ______

$\cos(\theta) \triangleq$ ______

$\tan(\theta) \triangleq$ ______ $= \frac{\sin(\theta)}{\cos(\theta)}$

A mnemonic for these definitions is "***S**oh**C**ah**T**oa*".

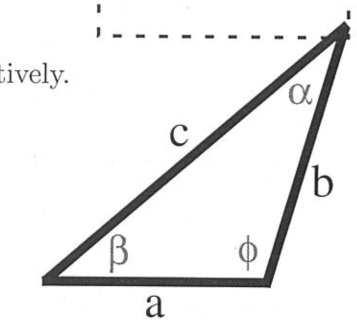
4.3 Pythagorean theorem and law of cosines - memorize. (Section 1.5.1).

Draw a right-triangle with a hypotenuse of length c and other sides of length a and b. Relate c^2 to a and b with the ***Pythagorean theorem***.
Result: $c^2 =$

A non-right-triangle has angles α, β, ϕ opposite sides a, b, c, respectively.
Use the ***law of cosines*** to complete each formula below.
Result: $c^2 = a^2 + b^2 - 2\,a\,b\cos(\phi)$

$a^2 =$

$b^2 =$

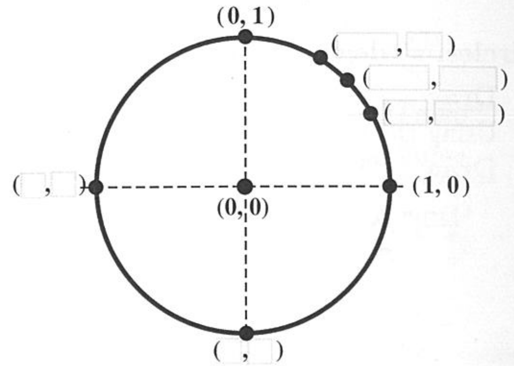


The ***Pythagorean theorem*** is a special case of the ***law of cosines***. **True/False**. (circle one).

4.4 Memorizing sine and cosine of common right-triangles.

Complete/memorize the following table. Label the coordinates of each point on the unit circle.

$\sin(0°)$ =	$\cos(0°)$ =
$\sin(30°)$ =	$\cos(30°)$ =
$\sin(45°)$ =	$\cos(45°)$ =
$\sin(60°)$ =	$\cos(60°)$ =
$\sin(90°)$ =	$\cos(90°)$ =

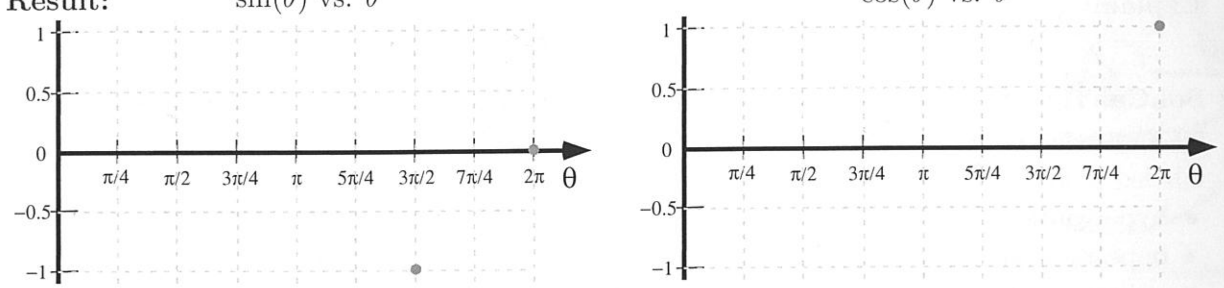


4.5 Graphing sine and cosine - (the now-obvious invention from "yesterday") (Section 1.5.2).

Graph sine and cosine as functions of the angle θ in radians over the range $0 \le \theta \le 2\pi$.

_____ was first to regard sine and cosine as **functions** (not just ratios of sides of a triangle) circa **1730**.

Result: $\sin(\theta)$ vs. θ $\cos(\theta)$ vs. θ



4.6 Ranges for arguments and return values for inverse trigonometric functions.

Determine all real return values and argument values for the following **real** trigonometric and inverse-trigonometric functions in computer languages such as Java, C, MotionGenesis, and MATLAB®.

Possible return values	Function	Possible argument values	Note
$\le z \le$	$z = \cos(x)$	$< x <$	
$\le z \le$	$z = \sin(x)$	$< x <$	
$-\infty < z < \infty$	$z = \tan(x)$	$-\infty < x < \infty$	$x \neq \frac{-\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2}, \ldots$
$\le z \le$	$z = \text{acos}(x)$	$\le x \le$	
$\le z \le$	$z = \text{asin}(x)$	$\le x \le$	
$-\pi/2 < z < \pi/2$	$z = \text{atan}(x)$	$-\infty < x < \infty$	
$< z \le$	$z = \text{atan2}(y, x)$	$< y <$ $< x <$	atan2(0, 0) is undefined

4.7 What is an angle? (Section 5.6).

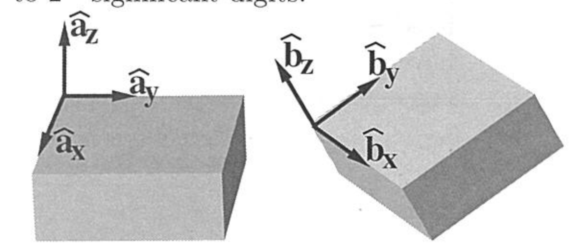
Draw the "geometry equipment" listed in the first column of the following table.
Complete the second column with appropriate ranges for the angle θ (in degrees).

Draw the "geometry equipment"	Appropriate range for θ
Two lines	$0° \le \theta \le$
Vector and line	$\le \theta \le$
Two vectors	$\le \theta \le$
Two vectors and a sense of positive rotation	$< \theta \le$
Two vectors, a sense of positive rotation, and time-history	$< \theta <$

4.8 Calculating dot-products, cross-products, and angles between vectors. (Section 5.4.3).

The following ${}^aR^b$ rotation table relates two sets of right-handed, orthogonal, unit vectors, namely $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{b}_x, \hat{b}_y, \hat{b}_z$. Perform the calculations below to 2^+ significant digits.

${}^aR^b$	$\hat{b}_x$	$\hat{b}_y$	$\hat{b}_z$
$\hat{a}_x$	0.9623	-0.0842	0.2588
$\hat{a}_y$	0.1701	0.9284	-0.3304
$\hat{a}_z$	-0.2125	0.3619	0.9077



(a) Efficiently determine the following dot-products.

$\hat{a}_x \cdot \hat{a}_x$ =	$\hat{a}_y \cdot \hat{a}_z$ =	$\hat{b}_z \cdot \hat{b}_y$ =
$\hat{a}_x \cdot \hat{b}_x$ =	$\hat{a}_x \cdot \hat{b}_y$ =	$\hat{b}_z \cdot \hat{a}_y$ =

(b) Determine the angles between the following vectors.

$\angle(\hat{a}_y, \hat{a}_y)$ = °	$\angle(\hat{b}_z, \hat{b}_x)$ = °
$\angle(\hat{a}_y, \hat{b}_y)$ = °	$\angle(\hat{b}_y, \hat{a}_z)$ = °

(c) Express the unit vector $\hat{u}$ in the direction of $3\,\hat{a}_z + 4\,\hat{b}_z$ as shown below.
Result:

$$\hat{u} = \_\_\_\_\_\ \hat{a}_z + \_\_\_\_\_\ \hat{b}_z$$

(d) Perform the following calculations involving $\vec{v}_1 = 2\,\hat{a}_x$ and $\vec{v}_2 = \hat{a}_x + \hat{b}_x$.
Result:

$\vec{v}_1 \cdot \vec{v}_2 =$ $\angle(\vec{v}_1, \vec{v}_2) =$ °

$\vec{v}_1 \times \vec{v}_2 = \_\_\_\_\_\ \hat{b}_y + \_\_\_\_\_\ \hat{b}_z = \_\_\_\_\_\ \hat{a}_y + \_\_\_\_\_\ \hat{a}_z$

(e) Express $\vec{v} = \hat{a}_y + \hat{b}_y$ in terms of $\hat{a}_x, \hat{a}_y, \hat{a}_z$.
Result:

$$\vec{v} = \_\_\_\_\_\ \hat{a}_x + \_\_\_\_\_\ \hat{a}_y + \_\_\_\_\_\ \hat{a}_z$$

4.9 Efficient calculation of the inverse of a rotation matrix. (Section 5.4.2).

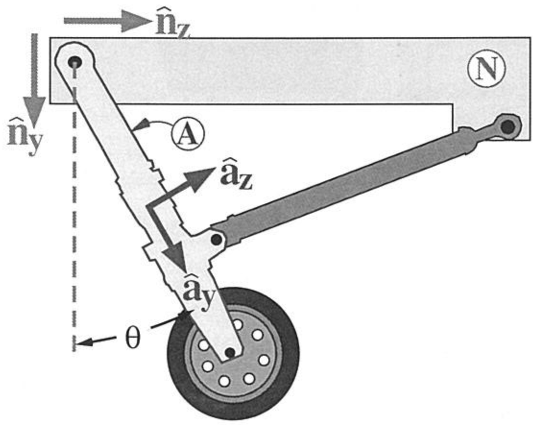
The following rotation matrix R relates two right-handed, orthogonal, unitary bases.
Calculate its inverse by-hand (no calculator) in less than 30 seconds.

$$R = \begin{bmatrix} 0.3830 & -0.6634 & 0.6428 \\ 0.9237 & 0.2795 & -0.2620 \\ -0.0058 & 0.6941 & 0.7198 \end{bmatrix} \quad \Rightarrow \quad R^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

4.10 SohCahToa: Rotation tables for a landing gear system. (Section 5.5).

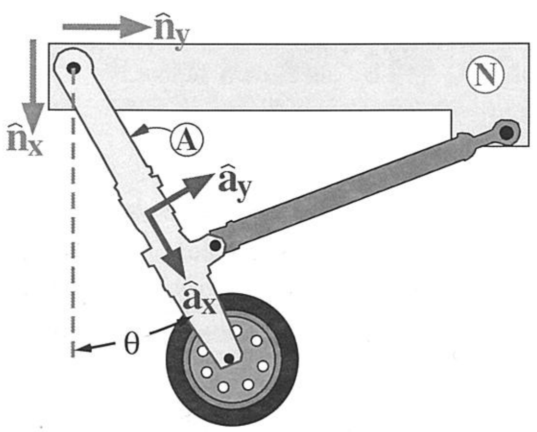
The figures below show three versions of the same landing gear system with a strut A that has a simple rotation relative to a fuselage N. In each figure, $\hat{\mathrm{n}}_\mathrm{x}$, $\hat{\mathrm{n}}_\mathrm{y}$, $\hat{\mathrm{n}}_\mathrm{z}$ is a set of orthogonal unit vectors fixed in N and $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ is a set of orthogonal unit vectors fixed in A. However, these unit vectors have a different orientation in each figure. **Redraw** the vectors $\hat{\mathrm{n}}_\mathrm{y}$ $\hat{\mathrm{n}}_\mathrm{z}$ and $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ so it is **easy to see** sines and cosines. Then, determine the ${}^aR^n$ rotation table for each figure.[1]

(a)



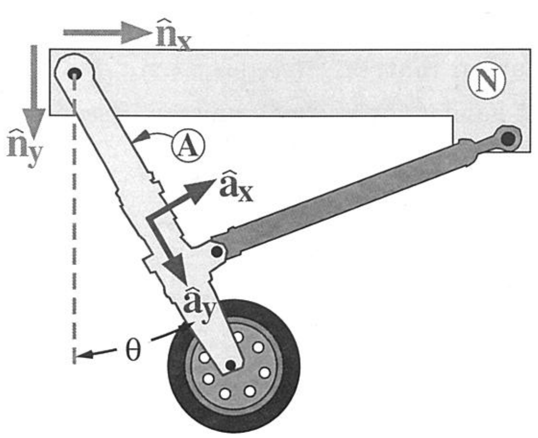
${}^aR^n$	$\hat{\mathrm{n}}_\mathrm{x}$	$\hat{\mathrm{n}}_\mathrm{y}$	$\hat{\mathrm{n}}_\mathrm{z}$
$\hat{\mathrm{a}}_\mathrm{x}$	1	0	0
$\hat{\mathrm{a}}_\mathrm{y}$	0	$\cos(\theta)$	$\sin(\theta)$
$\hat{\mathrm{a}}_\mathrm{z}$	0	$-\sin(\theta)$	$\cos(\theta)$

(b)



${}^aR^n$	$\hat{\mathrm{n}}_\mathrm{x}$	$\hat{\mathrm{n}}_\mathrm{y}$	$\hat{\mathrm{n}}_\mathrm{z}$
$\hat{\mathrm{a}}_\mathrm{x}$			
$\hat{\mathrm{a}}_\mathrm{y}$			
$\hat{\mathrm{a}}_\mathrm{z}$			

(c)

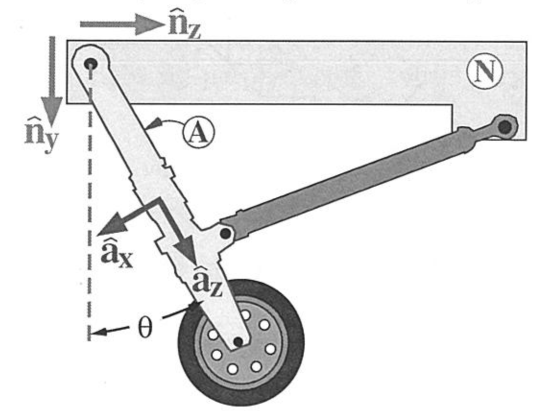


${}^aR^n$	$\hat{\mathrm{n}}_\mathrm{x}$	$\hat{\mathrm{n}}_\mathrm{y}$	$\hat{\mathrm{n}}_\mathrm{z}$
$\hat{\mathrm{a}}_\mathrm{x}$			
$\hat{\mathrm{a}}_\mathrm{y}$			
$\hat{\mathrm{a}}_\mathrm{z}$			

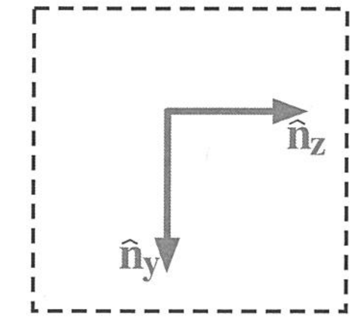
[1] Each figure has two missing vectors (e.g., $\hat{\mathrm{n}}_\mathrm{x}$ and $\hat{\mathrm{a}}_\mathrm{x}$ are missing from the first figure). Use the fact that each set of vectors is **right-handed** to add the missing vectors to each figure.

4.11 SohCahToa: Rotation table for a landing gear system - with disorderly unit vectors.

The following figure shows a landing gear system with a strut A that has a simple rotation relative to a fuselage N. Right-handed sets of orthogonal unit vectors $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ and $\hat{\mathrm{n}}_\mathrm{x}$, $\hat{\mathrm{n}}_\mathrm{y}$, $\hat{\mathrm{n}}_\mathrm{z}$ are fixed in A and N, respectively. θ is the angle from $\hat{\mathrm{n}}_\mathrm{y}$ to $\hat{\mathrm{a}}_\mathrm{z}$ with $+\hat{\mathrm{n}}_\mathrm{x}$ sense.



Redraw $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ in a geometrically suggestive way for forming the ${}^aR^n$ rotation matrix with sine and cosine.



Note: When $\theta = 0$, $\hat{\mathrm{a}}_\mathrm{x} \neq \hat{\mathrm{n}}_\mathrm{x}$ and $\hat{\mathrm{a}}_\mathrm{y} \neq \hat{\mathrm{n}}_\mathrm{y}$ and $\hat{\mathrm{a}}_\mathrm{z} \neq \hat{\mathrm{n}}_\mathrm{z}$.
Thus, $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ are "**disordered**" with $\hat{\mathrm{n}}_\mathrm{x}$, $\hat{\mathrm{n}}_\mathrm{y}$, $\hat{\mathrm{n}}_\mathrm{z}$.
Complete the blanks in the equations relating $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ to $\hat{\mathrm{n}}_\mathrm{x}$, $\hat{\mathrm{n}}_\mathrm{y}$, $\hat{\mathrm{n}}_\mathrm{z}$ and in the ${}^aR^n$ rotation table.

$\hat{\mathrm{a}}_\mathrm{x} =$ ______ $\hat{\mathrm{n}}_\mathrm{x}$ + ______ $\hat{\mathrm{n}}_\mathrm{y}$ + ______ $\hat{\mathrm{n}}_\mathrm{z}$

$\hat{\mathrm{a}}_\mathrm{y} =$ ______ $\hat{\mathrm{n}}_\mathrm{x}$ + ______ $\hat{\mathrm{n}}_\mathrm{y}$ + ______ $\hat{\mathrm{n}}_\mathrm{z}$

$\hat{\mathrm{a}}_\mathrm{z} =$ ______ $\hat{\mathrm{n}}_\mathrm{x}$ + ______ $\hat{\mathrm{n}}_\mathrm{y}$ + ______ $\hat{\mathrm{n}}_\mathrm{z}$

${}^aR^n$	$\hat{\mathrm{n}}_\mathrm{x}$	$\hat{\mathrm{n}}_\mathrm{y}$	$\hat{\mathrm{n}}_\mathrm{z}$
$\hat{\mathrm{a}}_\mathrm{x}$			
$\hat{\mathrm{a}}_\mathrm{y}$			
$\hat{\mathrm{a}}_\mathrm{z}$			

4.12 Rotation table concepts: What is an angle

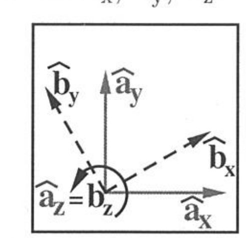
Given: Two sets of right-handed, orthogonal, unitary bases $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ and $\hat{\mathrm{b}}_\mathrm{x}$, $\hat{\mathrm{b}}_\mathrm{y}$, $\hat{\mathrm{b}}_\mathrm{z}$.

Question: Determine a numerical expression for each element of the 3×3 rotation table ${}^bR^a$ below so $\hat{\mathrm{b}}_\mathrm{z} = \hat{\mathrm{a}}_\mathrm{z}$ and the angle between $\hat{\mathrm{b}}_\mathrm{x}$ and $\hat{\mathrm{a}}_\mathrm{x}$ is 30°.
Draw $\hat{\mathrm{b}}_\mathrm{x}$, $\hat{\mathrm{b}}_\mathrm{y}$, $\hat{\mathrm{b}}_\mathrm{z}$ clearly show the relative orientation of the two bases.

Question: Are there other orientations of $\hat{\mathrm{b}}_\mathrm{x}$, $\hat{\mathrm{b}}_\mathrm{y}$, $\hat{\mathrm{b}}_\mathrm{z}$ and $\hat{\mathrm{a}}_\mathrm{x}$, $\hat{\mathrm{a}}_\mathrm{y}$, $\hat{\mathrm{a}}_\mathrm{z}$ such that $\hat{\mathrm{b}}_\mathrm{z} = \hat{\mathrm{a}}_\mathrm{z}$ and the angle between $\hat{\mathrm{b}}_\mathrm{x}$ and $\hat{\mathrm{a}}_\mathrm{x}$ is 30°? **Yes/No**.

Question: Is ${}^bR^a$ unique when $\hat{\mathrm{b}}_\mathrm{z} = \hat{\mathrm{a}}_\mathrm{z}$ and $\hat{\mathrm{b}}_\mathrm{x} \cdot \hat{\mathrm{a}}_\mathrm{x} = \frac{\sqrt{3}}{2}$? **Yes/No**.
If your answer is **No**, **draw** an alternative orientation for $\hat{\mathrm{b}}_\mathrm{x}$, $\hat{\mathrm{b}}_\mathrm{y}$, $\hat{\mathrm{b}}_\mathrm{z}$.

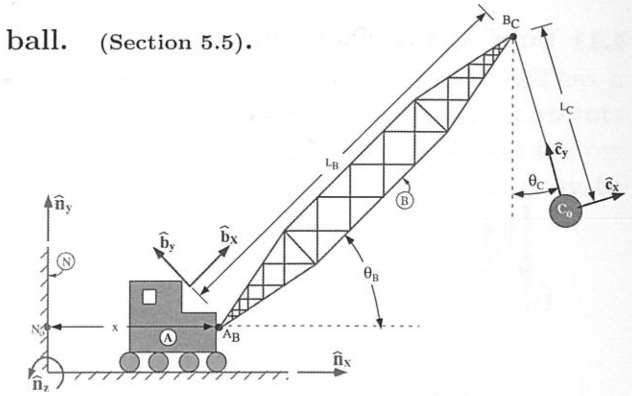
${}^bR^a$	$\hat{\mathrm{a}}_\mathrm{x}$	$\hat{\mathrm{a}}_\mathrm{y}$	$\hat{\mathrm{a}}_\mathrm{z}$
$\hat{\mathrm{b}}_\mathrm{x}$			
$\hat{\mathrm{b}}_\mathrm{y}$			
$\hat{\mathrm{b}}_\mathrm{z}$			



If "no", alternative

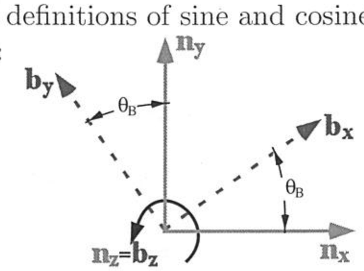
4.13 Rotation matrices for a crane and wrecking ball. (Section 5.5).

The figure to the right shows a crane whose cab A supports a boom B that swings a wrecking ball C. There are three sets of mutually perpendicular right-handed unit vectors, namely $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$; $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$; and $\widehat{\mathbf{c}}_x$, $\widehat{\mathbf{c}}_y$, $\widehat{\mathbf{c}}_z$. The point of this problem is to relate these sets of unit vectors.



Note: To relate the $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ unit vectors, it is helpful to redraw these vectors in a geometrically suggestive way as shown below.

(a) Use the definitions of sine and cosine to express each of $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ in terms of $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$.
Result:



$\widehat{\mathbf{b}}_x = \cos(\theta_B)\widehat{\mathbf{n}}_x + \sin(\theta_B)\widehat{\mathbf{n}}_y$

$\widehat{\mathbf{b}}_y =$

$\widehat{\mathbf{b}}_z =$

(b) Fill in the second and third rows of the ${}^bR^n$ *rotation table* shown to the right by extracting the various coefficients of the $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ unit vectors in the previous results.

${}^bR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{b}}_x$	$\cos(\theta_B)$	$\sin(\theta_B)$	0
$\widehat{\mathbf{b}}_y$			
$\widehat{\mathbf{b}}_z$			

(c) Form ${}^bR^n$, the ***rotation matrix*** relating $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ to $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$. Then form its transpose.
Result:

$$\begin{bmatrix}\widehat{\mathbf{b}}_x\\ \widehat{\mathbf{b}}_y\\ \widehat{\mathbf{b}}_z\end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}\widehat{\mathbf{n}}_x\\ \widehat{\mathbf{n}}_y\\ \widehat{\mathbf{n}}_z\end{bmatrix} \qquad \begin{bmatrix}\widehat{\mathbf{n}}_x\\ \widehat{\mathbf{n}}_y\\ \widehat{\mathbf{n}}_z\end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}\widehat{\mathbf{b}}_x\\ \widehat{\mathbf{b}}_y\\ \widehat{\mathbf{b}}_z\end{bmatrix}$$

(d) To relate the $\widehat{\mathbf{c}}_x$, $\widehat{\mathbf{c}}_y$, $\widehat{\mathbf{c}}_z$ and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ unit vectors, redraw these vectors in a geometrically suggestive way and then use the definitions of sine and cosine to express each of $\widehat{\mathbf{c}}_x$, $\widehat{\mathbf{c}}_y$, $\widehat{\mathbf{c}}_z$ in terms of $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$. Use these expressions to form the ${}^cR^n$ *rotation table*.
Result:

$\widehat{\mathbf{c}}_x =$

$\widehat{\mathbf{c}}_y =$

$\widehat{\mathbf{c}}_z =$

${}^cR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{c}}_x$			
$\widehat{\mathbf{c}}_y$			
$\widehat{\mathbf{c}}_z$			

(e) Use matrix multiplication to form the ${}^bR^c$ rotation table, i.e., ${}^bR^c = {}^bR^n * {}^nR^c$.
Simplify the results with the following trigonometric identities.

$\sin(\theta_B + \theta_C) = \sin(\theta_B)\cos(\theta_C) + \sin(\theta_C)\cos(\theta_B)$	$\cos(-\theta_C) = \cos(\theta_C)$
$\cos(\theta_B + \theta_C) = \cos(\theta_B)\cos(\theta_C) - \sin(\theta_B)\sin(\theta_C)$	$\sin(-\theta_C) = -\sin(\theta_C)$

Result:

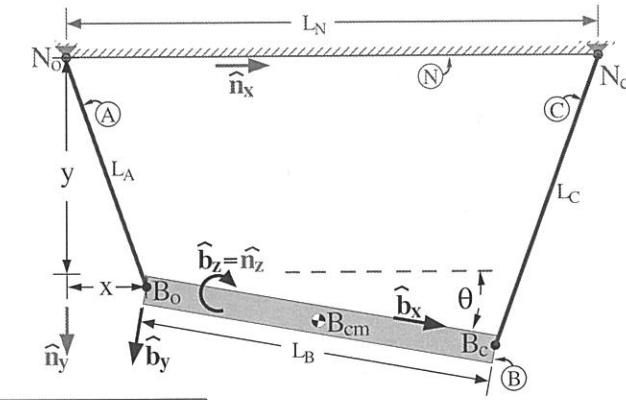
${}^bR^c$	$\widehat{\mathbf{c}}_x$	$\widehat{\mathbf{c}}_y$	$\widehat{\mathbf{c}}_z$
$\widehat{\mathbf{b}}_x$			
$\widehat{\mathbf{b}}_y$			
$\widehat{\mathbf{b}}_z$			

4.14 Varying cable lengths to position a construction hoist.

A uniform beam B is attached to a ceiling N by two variable-length cables (A and C).
Cable A attaches to the ceiling at point N_o of N and to the beam at point B_o of B.
Cable C attaches to the ceiling at point N_C of N and to the beam at point B_C of B.

Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in N and B, respectively, with $\widehat{\mathbf{b}}_z = \widehat{\mathbf{n}}_z$ perpendicular to the vertical plane containing points N_o, B_o, B_C, and N_C, and:

- $\widehat{\mathbf{n}}_x$ horizontally-right from N_o to N_C
- $\widehat{\mathbf{n}}_y$ vertically-**downward**
- $\widehat{\mathbf{b}}_x$ directed from B_o to B_C



Complete the ${}^bR^n$ rotation table.

${}^bR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{b}}_x$			
$\widehat{\mathbf{b}}_y$			
$\widehat{\mathbf{b}}_z$			

Description	Symbol	Type
Distance between N_o and N_C	L_N	Constant
Distance between B_o and B_C	L_B	Constant
$\widehat{\mathbf{n}}_x$ measure of B_o's position vector from N_o	x	Variable
$\widehat{\mathbf{n}}_y$ measure of B_o's position vector from N_o	y	Variable
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{b}}_x$ with $+\widehat{\mathbf{b}}_z$ sense	θ	Variable
Length of cable A (distance between N_o and B_o)	L_A	Variable
Length of cable C (distance between N_C and B_C)	L_C	Variable

Although planar geometry and the Pythagorean theorem can be used to calculate the cables' lengths, these techniques are less effective than vector methods and rotation matrices for more complicated geometry. To understand how to use vector methods and rotation matrices, proceed as follows.

(a) Using **only** the picture,[2] complete the following blanks in terms of x, y, L_B, L_N.
Result: B_o's position vector from N_o $\quad \vec{\mathbf{r}}^{B_o/N_o} = \_\_\_\,\widehat{\mathbf{n}}_x + \_\_\_\,\widehat{\mathbf{n}}_y$

B_C's position vector from N_C $\quad \vec{\mathbf{r}}^{B_C/N_C} = \_\_\_\_\_\_\,\widehat{\mathbf{n}}_x + \_\_\_\,\widehat{\mathbf{n}}_y + \_\_\_\,\widehat{\mathbf{b}}_x$

(b) An effective way to calculate each cable's ***length*** is with ***dot-products***. Use the following ***distance*** formulas (and the rotation matrix) to **efficiently** relate L_A^2 and L_C^2 to x, y, θ, L_N, L_B.
Result:

$$\vec{\mathbf{r}}^{B_o/N_o} \cdot \vec{\mathbf{r}}^{B_o/N_o} = L_A^2 = \boxed{x^2 + y^2}$$

$$\vec{\mathbf{r}}^{B_C/N_C} \cdot \vec{\mathbf{r}}^{B_C/N_C} = \qquad =$$

(c) **Implicitly** differentiate the previous equations to efficiently relate $\dot{L}_A$ and $\dot{L}_C$ to $\dot{x}$, $\dot{y}$, $\dot{\theta}$.
Result:

$$2\,L_A\,\dot{L}_A = \boxed{2\,(x\,\dot{x} + y\,\dot{y})}$$

$$2\,L_C\,\dot{L}_C =$$

(d) **Optional****: Using $L_N = 6$ m and $L_B = 4$ m, calculate L_A, L_C, $\dot{L}_A$, $\dot{L}_B$ when $x = 1$ m, $y = 2.5$ m, $\theta = 15°$, and $\dot{x} = 0$, $\dot{y} = 2\,\frac{\text{m}}{\text{s}}$, $\dot{\theta} = 0.2\,\frac{\text{rad}}{\text{sec}}$.

[2]**Hint**: To form $\vec{\mathbf{r}}^{B_C/N_C}$, use your finger to trace various paths to B_C from N_C.

Result: $L_A = 2.69$ m $\qquad L_C = 3.71$ m $\qquad \dot{L}_A = 1.9\ \frac{\text{m}}{\text{sec}} \qquad \dot{L}_C = 2.7\ \frac{\text{m}}{\text{sec}}$

4.15 Rotation matrices and angles. (Section 5.5).

Three sets of right-handed orthogonal, unitary bases $\widehat{\mathbf{a}}_x, \widehat{\mathbf{a}}_y, \widehat{\mathbf{a}}_z$, $\widehat{\mathbf{b}}_x, \widehat{\mathbf{b}}_y, \widehat{\mathbf{b}}_z$, and $\widehat{\mathbf{c}}_x, \widehat{\mathbf{c}}_y, \widehat{\mathbf{c}}_z$ and the ${}^aR^c$ and ${}^bR^c$ rotation matrices are given below.

${}^aR^c$	$\widehat{\mathbf{c}}_x$	$\widehat{\mathbf{c}}_y$	$\widehat{\mathbf{c}}_z$
$\widehat{\mathbf{a}}_x$	0.5	0.866	0
$\widehat{\mathbf{a}}_y$	-0.866	0.5	0
$\widehat{\mathbf{a}}_z$	0	0	1

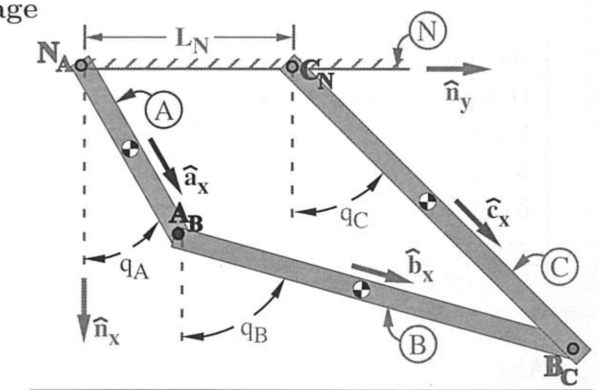
${}^bR^c$	$\widehat{\mathbf{c}}_x$	$\widehat{\mathbf{c}}_y$	$\widehat{\mathbf{c}}_z$
$\widehat{\mathbf{b}}_x$	$\cos(x)\cos(y)$	$\sin(x)\cos(y)$	$-\sin(y)$
$\widehat{\mathbf{b}}_y$	$-\sin(x)$	$\cos(x)$	0
$\widehat{\mathbf{b}}_z$	$\sin(y)\cos(x)$	$\sin(x)\sin(y)$	$\cos(y)$

Form an expression for the angle between $\widehat{\mathbf{a}}_x$ and the vector $\widehat{\mathbf{b}}_x + \widehat{\mathbf{c}}_x$ in terms of x and y.
Result:

$$\angle(\widehat{\mathbf{a}}_x,\ \widehat{\mathbf{b}}_x + \widehat{\mathbf{c}}_x) =$$

4.16 Configuration constraints for a four-bar linkage

Shown to the right is a planar four-bar linkage consisting of uniform rigid links A, B, and C and ground N. Link A is connected with revolute joints to N and B at points N_A and A_B, respectively. Link C is connected with revolute joints to N and B at points C_N and B_C, respectively.



Right-handed orthogonal unit vectors $\widehat{\mathbf{a}}_i$, $\widehat{\mathbf{b}}_i$, $\widehat{\mathbf{c}}_i$, and $\widehat{\mathbf{n}}_i$ ($i = $ x, y, z) are fixed in A, B, C, and N, with $\widehat{\mathbf{a}}_x$ directed from N_A to A_B, $\widehat{\mathbf{b}}_x$ from A_B to B_C, $\widehat{\mathbf{c}}_x$ from C_N to B_C, $\widehat{\mathbf{n}}_x$ vertically downward, $\widehat{\mathbf{n}}_y$ from N_A to C_N, and $\widehat{\mathbf{a}}_z = \widehat{\mathbf{b}}_z = \widehat{\mathbf{c}}_z = \widehat{\mathbf{n}}_z$ parallel to the axes of the revolute joints.

Quantity	Symbol	Value
Distance from N_A to A_B	L_A	1 m
Distance from A_B to B_C	L_B	2 m
Distance from B_C to C_N	L_C	2 m
Distance from C_N to N_A	L_N	1 m
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{a}}_x$	q_A	Variable
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{b}}_x$	q_B	Variable
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{c}}_x$	q_C	Variable

Create a vector "*loop equation*" using a sum of position vectors that start and end at point N_A.
Result:

$$L_A\,\widehat{\mathbf{a}}_x + \qquad + \qquad + \qquad = \vec{0}$$

Dot the loop equation with $\widehat{\mathbf{n}}_x$ and $\widehat{\mathbf{n}}_y$ to create two equations $f_i = 0$ ($i = $ x, y) that relate q_A, q_B, and q_C.[3] Next, Determine values of q_B and q_C that satisfy these two equations when $q_A = 30°$.

Result: Equations relating q_A, q_B, q_C.	Values when $q_A = 30°$
$f_1 = L_A * \cos(q_A) + L_B * \cos(q_B) - L_C * \cos(q_C)$	$q_B = 74.4775°$
$f_2 = \qquad + \qquad - L_C * \sin(q_C) - L_N$	$q_C = 45.5225°$

If $L_A < 1$ m, link A can be driven completely around, whereas if $L_A > 1$ m, it can only be driven 90°.

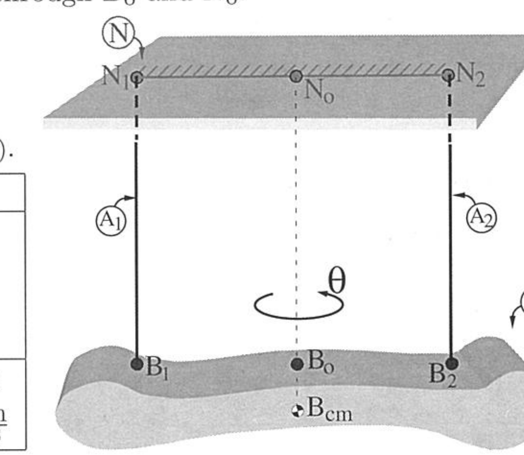
[3]Dot-products can be calculated by definition (inspection of the figure) or with rotation matrices.

4.17 Vertical displacement of a bifilar pendulum (useful for calculating moment of inertia).

Bifilar and trifilar pendulum are used to determine inertia properties of rigid bodies such as aircraft, spacecraft, and biological structures such as mass properties of humans. The following figure shows a rigid human bone B suspended by two rigid inextensible cables A_1 and A_2, each of which is attached to a flat horizontal ceiling N.

- Cable A_1 attaches to the ceiling at point N_1 of N and to the bone at point B_1 of B.
- Cable A_2 attaches to the ceiling at point N_2 of N and to the bone at point B_2 of B.
- Point N_o of N is centered between N_1 and N_2.
- Point B_o of B is centered between B_1 and B_2.
- Point B_{cm} (B's center of mass) and point B_o ***always*** lie directly below N_o.
- Initially, B_i lies directly below N_i (i=1, 2), respectively.
- B is rotated by an angle θ about the vertical line through B_o and N_o.

Relate y to L, h, and θ (defined in the following table).
Result: $\quad y^2 + \frac{1}{2}L^2\,[1 - \cos(\theta)] - h^2 = 0$

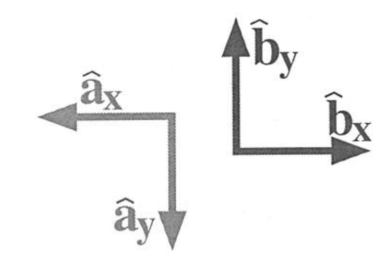


Calculate numerical values for y and $\dot{y}$ (3 significant digits).

Description	Symbol	Value
Distance between N_1 and N_2	L	1 m
Distance between N_i and B_i (i=1, 2)	h	1 m
B's rotation angle in N	θ	135°
B's rotation rate in N	$\dot{\theta}$	0.5 $\frac{\text{rad}}{\text{sec}}$
Distance between N_o and B_o	y	0.383 m
Time-derivative of y	$\dot{y}$	-0.231 $\frac{\text{m}}{\text{s}}$

4.18 3D visual thinking (draw/think 3D) - for disordered unit vectors

The figure to the right shows a right-handed orthogonal basis $\widehat{\mathbf{a}}_{xyz}$ consisting of unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ and another right-handed orthogonal basis $\widehat{\mathbf{b}}_{xyz}$ having unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.



The bases are initially oriented with $\theta = 0$ and $\widehat{\mathbf{b}}_x = -\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{b}}_y = -\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{b}}_z = \widehat{\mathbf{a}}_z$. Basis $\widehat{\mathbf{b}}_{xyz}$ is then subjected to a right-handed rotation relative to $\widehat{\mathbf{a}}_{xyz}$ in one of two ways, as described below.
Express each ${}^bR^a$ rotation matrix below in terms of θ.

Rotation of $\widehat{\mathbf{b}}_{xyz}$ in $\widehat{\mathbf{a}}_{xyz}$ characterized by $+\theta\,\widehat{\mathbf{a}}_z$
(θ is the angle from $-\widehat{\mathbf{a}}_x$ to $\widehat{\mathbf{b}}_x$ with $+\widehat{\mathbf{a}}_z$ sense)

${}^bR^a$	$\widehat{\mathbf{a}}_x$	$\widehat{\mathbf{a}}_y$	$\widehat{\mathbf{a}}_z$
$\widehat{\mathbf{b}}_x$			
$\widehat{\mathbf{b}}_y$			
$\widehat{\mathbf{b}}_z$			

Rotation of $\widehat{\mathbf{b}}_{xyz}$ in $\widehat{\mathbf{a}}_{xyz}$ characterized by $+\theta\,\widehat{\mathbf{a}}_y$
(θ is the angle from $\widehat{\mathbf{a}}_z$ to $\widehat{\mathbf{b}}_z$ with $+\widehat{\mathbf{a}}_y$ sense)

${}^bR^a$	$\widehat{\mathbf{a}}_x$	$\widehat{\mathbf{a}}_y$	$\widehat{\mathbf{a}}_z$
$\widehat{\mathbf{b}}_x$			
$\widehat{\mathbf{b}}_y$			
$\widehat{\mathbf{b}}_z$			

5.1 Notations for derivatives. (Section 1.6.1).

Date	Person	Symbols for 1^{st}, 2^{nd}, and 3^{rd} derivatives		
1675		$\frac{dy}{dt}$	$\frac{d^2y}{dt^2}$	$\frac{d^3y}{dt^3}$
1675		$\dot{y}$	$\ddot{y}$	$\dddot{y}$
1770	(trained by Euler)	y'	y''	y'''
1850	Cauchy/Weierstrauss	$\lim\limits_{h \to 0} \frac{y(t+h) - y(t)}{h}$	?	?
1786	Legendre (introduced partials then abandoned)	$\frac{\partial y}{\partial x}$	$\frac{\partial^2 y}{\partial x^2}$	$\frac{\partial^3 y}{\partial x^3}$
1841	Jacobi (re-introduced partials again)			

There was bitter rivalry between Newton and Leibniz, and the notations of Leibniz and Newton are not entangled.
For example, $\frac{d\dot{y}}{dt}$ is written in Leibniz's notation as or Newton's as .

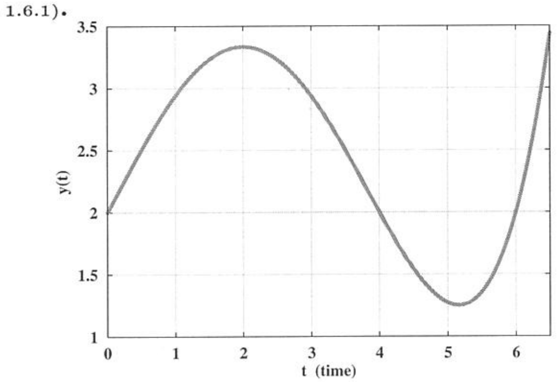
5.2 Leibniz's shorthand notation for 3^{rd} derivatives

Write the explicit expression for the following 3^{rd} derivative (so it only contains 1^{st} derivatives).

Result:

$$\frac{d^3y}{dt^3} \triangleq$$

5.3 Geometric interpretation of a derivative. (Section 1.6.1).



Estimate the 1^{st}-derivative of the function $y(t)$ shown to the right at $t = 0,\ 2,\ 4,\ 6$.

Pick your answers from: **-1**, **0**, **1**, **2**.

Result: $\left.\frac{dy}{dt}\right|_{t=0} =$ $\quad\left.\frac{dy}{dt}\right|_{t=2} =$

$\left.\frac{dy}{dt}\right|_{t=4} =$ $\quad\left.\frac{dy}{dt}\right|_{t=6} =$

Estimate the <u>sign</u> of the 2^{nd}-derivative of $y(t)$ from the answers **−**, **0**, or **+**.
Answer **0** when the absolute value of the 2^{nd}-derivative is estimated to be less than 0.5.

Result: $\left.\frac{d^2y}{dt^2}\right|_{t=0}$ is $\quad\left.\frac{d^2y}{dt^2}\right|_{t=2}$ is $\quad\left.\frac{d^2y}{dt^2}\right|_{t=4}$ is $\quad\left.\frac{d^2y}{dt^2}\right|_{t=6}$ is

5.4 Derivatives of commonly-encountered functions. (Section 1.6.5).

Differentiate the following functions that depend on t (time). Ensure answers involving x are valid when x is either constant or depends on time, e.g., when $x = t^3$.

Result:

$\frac{d}{dt}\, t^2 =$ $\quad\frac{d}{dt}\, t^3 =$ $\quad\frac{d}{dt}\, t^{47} =$

$\frac{d}{dt}\, \sin(t) =$ $\quad\frac{d}{dt}\, \cos(t) =$ $\quad\frac{d}{dt}\, \cos(x) =$

$\frac{d}{dt}\, e^t =$ $\quad\frac{d}{dt}\, \ln(t) =$ $\quad\frac{d}{dt}\, \ln(x) =$

5.5 Good product rule for differentiation. (Section 1.6.7).

The ***good product rule for differentiation*** that works when u and v are scalars, vectors, or matrices is (circle the correct answer):

$$\frac{d(u*v)}{dt} = \frac{du}{dt} * v + u * \frac{dv}{dt} \qquad \frac{d(u*v)}{dt} = u * \frac{dv}{dt} + v * \frac{du}{dt} \qquad \frac{d(u*v)}{dt} = v * \frac{du}{dt} + u * \frac{dv}{dt}$$

5.6 Differentiating quotients: Use the product rule and exponents. **(Division - "Just say No").**
Although the "***quotient rule***" can be used to calculate the derivative with respect to t of the ratio of two functions $\frac{f(t)}{g(t)}$, it can be easier to rewrite the ratio as $f(t) * g(t)^{-1}$ then use the ***product rule***. Use this idea to first rewrite the following ratio of two functions as a product and then use the ***product rule*** to calculate its derivative.
Result: $\frac{\ln(t)}{t^2} =$ $\qquad$ $\frac{d}{dt}[\ln(t) / t^2] =$

5.7 Example of the "good product rule" for differentiation. **(Should take less than 2 minutes).**
The "good" product rule is easy-to-use for ***very quickly*** differentiating complex expressions. Knowing x and y are variables that depend on the independent variable t (time), determine the ordinary time-derivative of the function f when[1]

$$f(t) = \sin(t) * \cos(x+y) * (\dot{x})^2 * e^t * \ln(y) / x$$

Result:

$$\begin{aligned} \frac{df}{dt} &= \cos(t) * \cos(x+y) * (\dot{x})^2 * e^t * \ln(y) / x \\ &- \sin(t) * \sin(x+y) * (\dot{x}+\dot{y}) * (\dot{x})^2 * e^t * \ln(y) / x \\ &+ \\ &+ \\ &+ \\ &- \end{aligned}$$

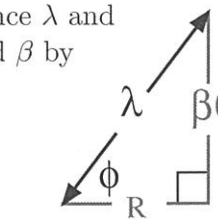
5.8 Differentiation concepts. **(Section 1.6.10).**
The following equation relates the dependent variable y to the independent variable t.

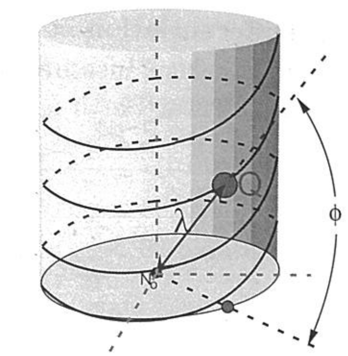
$$y^4 - 8y = 3t^2 + \sin(t)$$

Find a general expression for the ordinary derivative $\frac{dy}{dt}$ in terms of t and y.
Find a **numerical** value for $\frac{dy}{dt}$ at $t = 0$ when y is **positive**.
Hint: The value of y is not arbitrary. If you encounter difficulty, consider ***implicit differentiation***..
Result: $\frac{dy}{dt} =$ $\qquad$ $\left.\frac{dy}{dt}\right|_{t=0} =$

[1]Symbols for the 1^{st} and 2^{nd} ordinary time-derivatives of x include $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ (introduced by ***Leibniz***), $\dot{x}$ and $\ddot{x}$ (introduced by ***Newton***), and `x'` and `x''` (introduced by ***Lagrange*** and used by MotionGenesis).

5.9 Review of explicit and implicit differentiation. **(Section 1.6.10).**
The figure to the right shows a point Q on a cylindrical helix. Two geometrically significant quantities are a distance λ and an angle ϕ that are related to two **constants** R and β by





$$\lambda^2 = R^2 + (\beta\,\theta)^2 \qquad \tan(\phi) = \frac{\beta\,\theta}{R}$$

Determine $\dot{\lambda}$ and $\dot{\phi}$ (the time-derivatives of λ and ϕ) in terms of θ, $\dot{\theta}$, R, β, etc., using the two methods described below.

(a) ***Explicit differentiation***
Solve explicitly for λ and ϕ and then differentiate the resulting expression.
Result:
$\lambda = \sqrt{R^2 + (\beta\,\theta)^2}$ $\qquad$ $\phi = \text{atan}(\frac{\beta\,\theta}{R})$
$\dot{\lambda} =$ $\qquad$ $\dot{\phi} =$

(b) ***Implicit differentiation***
Differentiate the equations involving λ^2 and $\tan(\phi)$ and then solve for $\dot{\lambda}$ and $\dot{\phi}$.
Result:
$\dot{\lambda} =$ $\qquad$ $\dot{\phi} =$ $\qquad = \frac{\beta\,R}{\lambda^2}\,\dot{\theta}$

(c) **Explicit/Implicit** differentiation of λ is easier and computationally more efficient.

5.10 Review of partial and ordinary differentiation. **(Section 1.6.2).**
The kinetic energy K of the system to the right can be written in terms of constants m^A, m^Q, L and time-dependent variables x, θ, as

$$K = \frac{1}{2}m^A\,\dot{x}^2 + \frac{1}{2}m^Q\,[\dot{x}^2 + L^2\,\dot{\theta}^2 + 2\,L\,\cos(\theta)\,\dot{x}\,\dot{\theta}]$$

Use partial and ordinary differentiation to form the following ingredients for ***Lagrange's equations of motion***.

$\frac{\partial K}{\partial \theta} =$ $\qquad$ $\frac{\partial K}{\partial \dot{\theta}} =$ $\qquad$ $\frac{d}{dt}\frac{\partial K}{\partial \dot{\theta}} =$

5.11 Differentiation concepts: What is dt? **(Section 1.6.3).**
A continuous function $z(t)$ depends on $x(t)$, $y(t)$, and time t as $\qquad z = x + y^2 \sin(t)$
At a certain instant of time, $y = 1$ and z simplifies to $\qquad z = x + \sin(t)$
Find the time-derivative of z at the instant when $y = 1$.
Result: $\left.\frac{dz}{dt}\right|_{y=1} =$

5.12 Differentiation concepts. (Section 1.6.3).
The scalar v measures a baseball's upward-velocity. Knowing $v = 0$ when the ball reaches maximum height near Earth ($g \approx 9.8 \frac{\text{m}}{\text{s}^2}$), decide if the following statement about v's time derivative is true.

$$\frac{dv}{dt} = \frac{d(0)}{dt} = 0 \qquad \text{True/False}$$

Explain:

5.13 Integrals of commonly-encountered functions. (Section 1.7).
Calculate the following indefinite integrals in terms of an indefinite constant C (regard t as positive).
Result:

$\int t^2\, dt =$ $\qquad$ $\int t^3\, dt =$ $\qquad$ $\int t^8\, dt =$

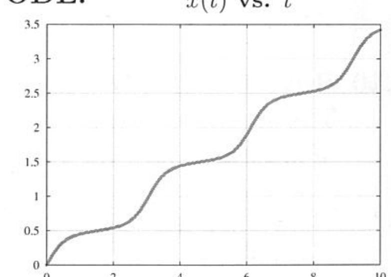
$\int t^{-3}\, dt =$ $\qquad$ $\int t^{-2}\, dt =$ $\qquad$ $\int t^{-1}\, dt =$

$\int \sin(t)\, dt =$ $\qquad$ $\int \cos(t)\, dt =$ $\qquad$ $\int e^t\, dt =$

$\int 5\, dt =$ $\qquad$ $\int 5/t\, dt =$ $\qquad$ $\int (5 + \frac{1}{t})\, dt =$

5.14 Optional: † Continuous numerical solution of a nonlinear ODE.**

$x(t)$ vs. t



Plot the continuous solution $x(t)$ to the following ordinary differential equation for $0 \le t \le 10$ with data every 0.2 sec. Use an initial value $x(0) = 0$ and use the initial value of $\dot{x}$ that is closest to 1.

$$\sin(\dot{x}) + 4\,\dot{x}^2 - 1.9\cos(2\,\pi\,x) - 2 = 0$$

Hint: A "clever" way to solve this **nonlinear** ODE for $x(t)$ is

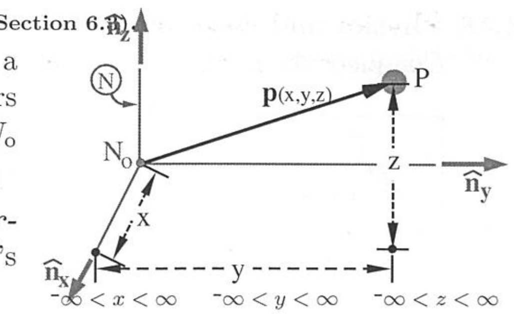
- Use the given equation and initial value $x(0) = 0$ to solve for $\dot{x}$ at $t{=}0$.
  For example, the technique in Section 1.10 finds $\dot{x}(t{=}0) \approx 0.8841161$ when $x(t{=}0) = 0$.
- Time-differentiate the 1^{st}-order ODE that is **nonlinear** in $\dot{x}$ to form a 2^{nd}-order ODE that is **linear** in $\ddot{x}$.
  Then, solve the 2^{nd}-order ODE for $\ddot{x}$.

$$\cos(\dot{x})\,\ddot{x} + 8\,\dot{x}\,\ddot{x} + 3.8\,\pi\,\sin(2\,\pi\,x)\,\dot{x} = 0 \quad \Rightarrow \quad \ddot{x} = \frac{-3.8\,\pi\,\sin(2\,\pi\,x)\,\dot{x}}{\cos(\dot{x}) + 8\,\dot{x}}$$

- Numerically integrate the 2^{nd}-order ODE with the initial values of $x(0)$ and $\dot{x}(0)$

5.15 Vector differentiation and Cartesian coordinates. (Section 6.3).
The figure to the right shows a baseball P moving in a reference frame N. Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ are fixed in N as shown. P's location from point N_o (a point fixed in N) can be specified with three ***coordinates***.

A ***Cartesian coordinate system*** locates P with the ***coordinates*** x, y, z, which are the $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ measures of P's position vector from N_o.



(a) Report the time-derivative in N of $\widehat{\mathbf{n}}_x$ and briefly justify your answer.
Result:

$$\frac{{}^N d\widehat{\mathbf{n}}_x}{dt} = \qquad \begin{array}{l} \widehat{\mathbf{n}}_x\text{'s magnitude} \\ \widehat{\mathbf{n}}_x\text{'s direction} \end{array}$$

(b) Express P's position vector from N_o in terms of x, y, z, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$.
Using the definition of the derivative of a vector [equation (6.3)] and the product rule for vector derivatives, find the time-derivative in N of $\vec{\mathbf{p}}$ and express it in terms of $\dot{x}$, $\dot{y}$, $\dot{z}$, and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$.[2]
Result:

$$\vec{\mathbf{p}} = \qquad + y\,\widehat{\mathbf{n}}_y + z\,\widehat{\mathbf{n}}_z \qquad\qquad \frac{{}^N d\vec{\mathbf{p}}}{dt} = \dot{x}\,\widehat{\mathbf{n}}_x + \qquad \widehat{\mathbf{n}}_y + \qquad \widehat{\mathbf{n}}_z$$

5.16 Vector differentiation and reference frames. (Section 6.3).
The following vectors are expressed in terms of the orthogonal unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ and t time. Circle the vectors that can be differentiated without consideration of a reference frame.

$\vec{0}$ $\qquad$ $2\,\widehat{\mathbf{a}}_x + 4\,\widehat{\mathbf{a}}_y$ $\qquad$ $2\,\widehat{\mathbf{a}}_x + t\,\widehat{\mathbf{a}}_y$

$\widehat{\mathbf{a}}_x$ $\qquad$ $2\,\widehat{\mathbf{a}}_x + 4\,\widehat{\mathbf{a}}_y + 6\,\widehat{\mathbf{a}}_z$ $\qquad$ $2\,\widehat{\mathbf{a}}_x + t\,\widehat{\mathbf{a}}_y + \sin(t)\,\widehat{\mathbf{a}}_z$

5.17 Textbook definitions of vector differentiation.
A vector has magnitude and direction. The change of a vector's magnitude relates to scalar differentiation. The change of a vector's direction depends on **reference frame**. The first notation that explicitly showed dependence of a ***vector derivative*** on a ***reference frame*** was introduced in 1950 by the preeminent dynamicist Thomas Kane who taught that a mathematical ***definition*** should:

- Involve ingredients that themselves are reasonably understood and/or defined.
  In other words, the definition is comprehensible to the intended audience.
- Be useful for directly or indirectly proving all other related properties.

Report one or more definitions for the derivative of a vector from textbooks (e.g., undergraduate/graduate physics or engineering textbooks) and/or from the Internet and determine if both the ***definition*** and ***notation*** clearly shows that a vector's derivative depends on reference frame.

[2]The variables x, y, and z implicitly depend on time t.
Leibniz, ***Newton***, and ***Lagrange*** introduced the symbols $\frac{dx}{dt}$, $\dot{x}$, and x', respectively, to denote the time-derivative of x.

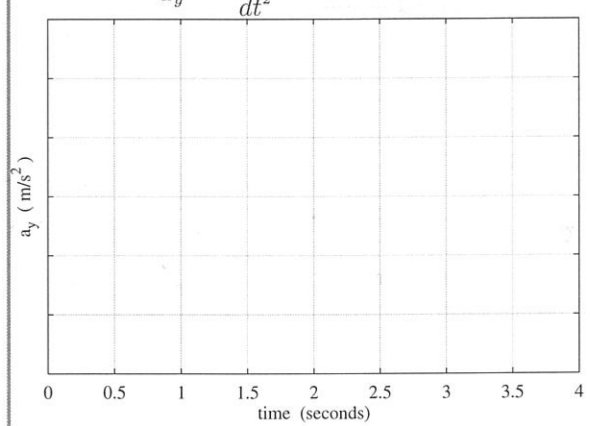
5.18 Physics and calculus: Graphing $\vec{F} = m\vec{a}$ for a sky-diver and rocket-sled.

Complete the missing statements, axes values, and graphs. Use Earth's gravitational acceleration $g \approx 10 \, \frac{m}{s^2}$.

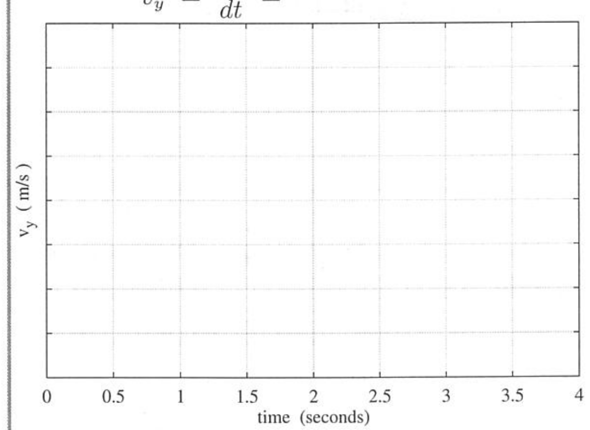
A sky-diver free-falls for 4 seconds after leaving a stationary helicopter from a height $y = 200$ m above Earth (y is positive-upward).

The only relevant force is Earth's gravity.

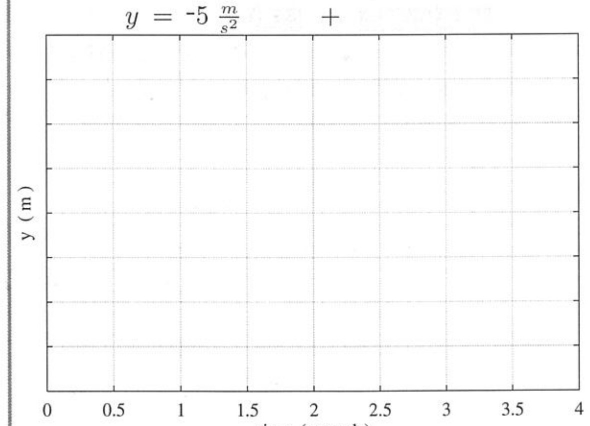
$a_y \triangleq \frac{d^2y}{dt^2} =$



$v_y \triangleq \frac{dy}{dt} =$

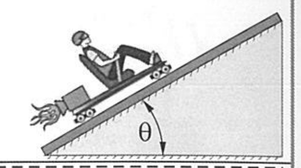


$y =$ -5 $\frac{m}{s^2}$ $+$



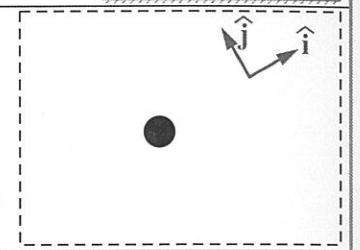
A rocket-sled of mass m is thrust along smooth inclined rails with time-varying force F_T.

The variable x measure's the sled's position along the rails. Initially, $x = 0$ and $\dot{x} = 0$.



FBD. Draw forces

Below: Form $\vec{F}_{\text{Net}}$ and then set $\vec{F}_{\text{Net}} = m\vec{a}$. Use symbols m, g, F_T, F_N, θ.



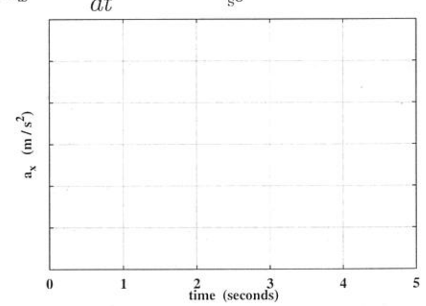
$\hat{\imath} + \qquad \hat{\jmath} = m \qquad \hat{\imath}$

$\hat{\imath}$: $\qquad = \qquad \Rightarrow \frac{d^2x}{dt^2} = \qquad -$

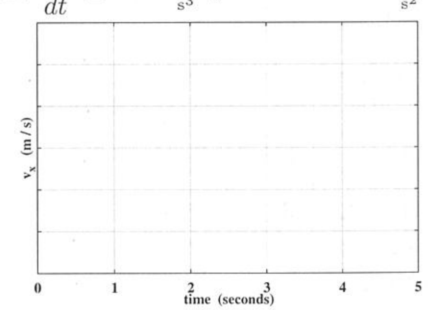
$\hat{\jmath}$: $\qquad = \qquad \Rightarrow F_N =$

Use $\theta = 30°$, $m = 100$ kg, $F_T = 600 \, \frac{N}{s} * t$ for the following.

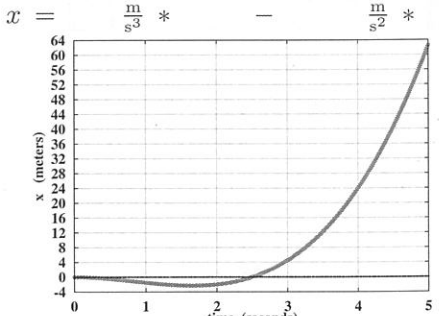
$a_x \triangleq \frac{dv_x}{dt} = \qquad \frac{m}{s^3} * \qquad - \qquad \frac{m}{s^2}$



$v_x \triangleq \frac{dx}{dt} = \qquad \frac{m}{s^3} * \qquad - \qquad \frac{m}{s^2} *$



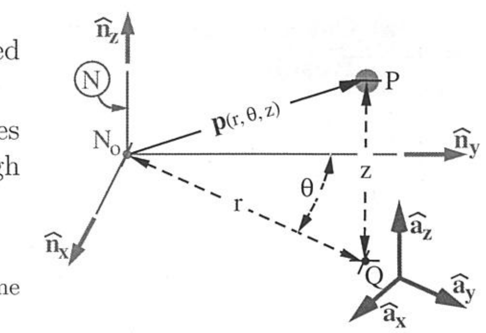
$x = \qquad \frac{m}{s^3} * \qquad - \qquad \frac{m}{s^2} *$



5.19 Cylindrical coordinates, position, and orientation

The following figure shows a baseball P moving in a reference frame N. Right-handed orthogonal unit vectors $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ are fixed in N as shown.

P's location from point N_o (a point fixed in N) can be specified with ***cylindrical coordinates*** consisting of:



- r, the distance between N_o and the point Q that traces out the projection of P onto the plane that passes through N_o and is perpendicular to $\hat{n}_z$
- θ, the $^+\hat{n}_z$ measure of the angle from $\hat{a}_y$ to $\hat{n}_y$.
  Note: Orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are fixed in a reference frame A with $\hat{a}_y$ pointing from N_o to Q, $\hat{a}_z = \hat{n}_z$, and $\hat{a}_x = \hat{a}_y \times \hat{a}_z$.
- z, the $^+\hat{n}_z$ measure of P's position vector from N_o.

$r \geq 0 \qquad -\pi \leq \theta \leq \pi \qquad -\infty < z < \infty$

(a) The magnitude of $\hat{a}_y$ **changes/stays constant** (circle one) with time.
The direction of $\hat{a}_y$ in N **changes/stays constant** with time.

(b) To relate the $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ and $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ unit vectors, **redraw** these vectors in a geometrically suggestive way and then form the ${}^aR^n$ rotation table that relates $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ to $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$.
Result:

${}^aR^n$	$\hat{n}_x$	$\hat{n}_y$	$\hat{n}_z$
$\hat{a}_x$			
$\hat{a}_y$			
$\hat{a}_z$			

(c) By **inspection**, express $\vec{p}$ (P's position vector from N_o) in terms of r, θ, z, and $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ and then use the rotation table to express $\vec{p}$ in terms of r, θ, z, and $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$.
Result: $\vec{p} = r\,\hat{a}_y + \qquad \hat{a}_z \qquad\qquad \vec{p} = r\,\sin(\theta)\,\hat{n}_x + \qquad \hat{n}_y + z\,\hat{n}_z$

In view of these expressions for $\vec{p}$, it is clear that $\vec{p}$ is a vector function of $r/\theta/z$ [circle the correct variable(s)] in A whereas $\vec{p}$ is a vector function of $r/\theta/z$ in N.

(d) Alternately, the ***Cartesian coordinates*** x, y, z locate P from N_o as $\vec{p} = x\,\hat{n}_x + y\,\hat{n}_y + z\,\hat{n}_z$.
Express x and y in terms of r and θ. Then, express r and θ in terms of x and y.
Result: [Note: The `atan2` function is described in Section 1.5.4 and is undefined if $x = y = 0$.]

$x = r\,\sin(\theta) \qquad\qquad y =$

$r = \sqrt{\qquad + \qquad} \qquad\qquad \theta = \texttt{atan2}(x,\,y)$ not $\texttt{atan2}(y,\,x)$

(e) The location of P is **uniquely** defined by r, θ, z. **True/False**.
The values of r, θ, z are **uniquely** defined by the location of P. **True/False**.

(f) The variables r and θ may be used to describe the motion of a particle P that is constrained to a flat horizontal circular plate. What location of P would cause θ to be undefined?
Result:

5.20 Cylindrical coordinates and vector differentiation via definition. (Section 6.3).

Referring to Homework 5.19, use the definition of a vector derivative [equation (6.3)] to find the time-derivative in N of $\vec{p}$ and express it in terms of r, θ, z, $\dot{r}$, $\dot{\theta}$, $\dot{z}$ and $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$. Then, use the rotation table to re-express your result in terms of $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.

Result: $\frac{{}^N d\vec{p}}{dt} = \Big[\qquad \Big]\,\hat{n}_x + \Big[\qquad \Big]\,\hat{n}_y + \qquad \hat{n}_z$

$= r\,\dot{\theta}\,\hat{a}_x + \dot{r}\,\hat{a}_y + \dot{z}\,\hat{a}_z$

The expression for $\frac{{}^N d\vec{p}}{dt}$ is simpler when it is expressed in terms of $(\hat{n}_x, \hat{n}_y, \hat{n}_z)$ / $(\hat{a}_x, \hat{a}_y, \hat{a}_z)$.

5.21 Cylindrical coordinates and vector differentiation via angular velocity

This problem introduces an efficient method for differentiating the vector $\vec{p}$ of Homework 5.20.

(a) Calculate the time-derivative in A of $\vec{p}$, and express it in terms of $\dot{r}$, $\dot{\theta}$, $\dot{z}$ and $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.

Result:
$$\vec{p} = r\,\hat{a}_y + z\,\hat{a}_z \qquad \Rightarrow \qquad \frac{{}^A d\vec{p}}{dt} = \dot{r}\,\hat{a}_y + \dot{z}\,\hat{a}_z$$

(b) $\frac{{}^N d\vec{p}}{dt}$ $\qquad$ $\frac{{}^A d\vec{p}}{dt}$ (refer to your results in Homework 5.20 and fill in the blank with = or $\neq$).

(c) When the orientation of a reference frame A in a reference frame N changes in such a way that there exists a unit vector whose orientation in both A and N is independent of time t, then A is said to have a ***simple angular velocity*** in N. Referring to the figure in Homework 5.19, find two unit vectors that are ***fixed*** in both A and N.

Result: and

(d) Calculate $\frac{{}^N d\vec{p}}{dt}$ with the ***golden rule for vector differentiation*** (show below) in conjunction with the fact that A's angular velocity in N is ${}^N\vec{\omega}^A = \text{-}\dot{\theta}\,\hat{a}_z$.

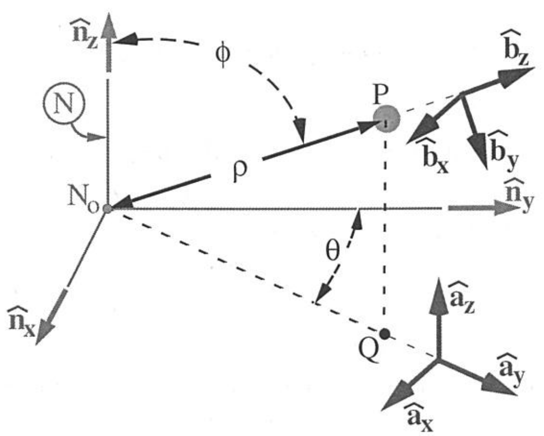
Result:
$$\frac{{}^N d\vec{p}}{dt} = \frac{{}^A d\vec{p}}{dt} + {}^N\vec{\omega}^A \times \vec{p} = \qquad \hat{a}_x + \qquad \hat{a}_y + \dot{z}\,\hat{a}_z$$

(e) Compared to the definition of vector differentiation in Homework 5.20, the ***golden rule for vector differentiation*** is an **easier/harder** way to calculate $\frac{{}^N d\vec{p}}{dt}$.

5.22 Spherical coordinates, position, and orientation

The following figure shows a baseball P moving in a reference frame N. Right-handed orthogonal unit vectors $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ are fixed in N as shown. P's location from point N_o (a point fixed in N) can be specified with ***spherical coordinates***, consisting of:[3]

- ρ, the distance between N_o and P.
- θ, the $+\hat{n}_z$ measure of the angle <u>from</u> $\hat{a}_y$ <u>to</u> $\hat{n}_y$, where $\hat{a}_y$ points from N_o to point Q. Point Q traces out the projection of P onto the plane perpendicular to $\hat{n}_z$ and passing through N_o.
- ϕ, the angle <u>between</u> $\hat{n}_z$ and P's position vector from N_o.



$\rho \geq 0 \qquad -\pi \leq \theta \leq \pi \qquad 0 \leq \phi \leq \pi$

Right-handed sets of orthogonal unit vectors $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ and $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ are fixed in reference frames A and B, respectively.

- $\hat{b}_z$ points from N_o to P
- $\hat{b}_x = \hat{a}_x$
- $\hat{b}_y = \hat{b}_z \times \hat{b}_x$

- $\hat{a}_y$ points from N_o to Q
- $\hat{a}_z$ points vertically upward ($\hat{a}_z = \hat{n}_z$)
- $\hat{a}_x = \hat{a}_y \times \hat{a}_z$

(a) The magnitude of $\hat{b}_z$ **changes/stays constant** with time.
The direction of $\hat{b}_z$ in N **changes/stays constant** with time.
The direction of $\hat{a}_z$ in N **changes/stays constant** with time.

[3]Assume that ρ, θ, ϕ are variables that depend on time t.

(b) <u>**Draw**</u> $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ and $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ in a geometrically helpful way to form the ${}^aR^n$ rotation table. Similarly, <u>draw</u> $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ and $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ to facilitate forming the ${}^bR^a$ rotation table. Next, use matrix multiplication to form the ${}^bR^n$ rotation tables.

Attempt to draw a 3D picture to form the ${}^bR^n$ rotation table directly from the picture and geometry. It is **easier/harder** (circle one) to multiply matrices to form the ${}^bR^n$ table rather than to draw a 3D picture and use geometry to form ${}^bR^n$.

Result:

${}^aR^n$	$\hat{n}_x$	$\hat{n}_y$	$\hat{n}_z$
$\hat{a}_x$			
$\hat{a}_y$			
$\hat{a}_z$			

${}^bR^a$	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{b}_x$			
$\hat{b}_y$			
$\hat{b}_z$			

${}^bR^n$	$\hat{n}_x$	$\hat{n}_y$	$\hat{n}_z$
$\hat{b}_x$			
$\hat{b}_y$	$\sin(\theta)\cos(\phi)$		
$\hat{b}_z$	$\sin(\theta)\sin(\phi)$	$\cos(\theta)\sin(\phi)$	

(c) By **inspection**, express $\vec{p}$ (the position of P from point N_o) in terms of ρ, θ, ϕ, and $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$ and then use the rotation table to express $\vec{p}$ in terms of ρ, θ, ϕ, and $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$.

Result: $\vec{p} = \rho\,\hat{b}_z \qquad \vec{p} = \qquad \hat{n}_x + \qquad \hat{n}_y + \rho\cos(\phi)\,\hat{n}_z$

(d) In view of your previous expressions for $\vec{p}$, it is clear that $\vec{p}$ is a vector function of $\rho/\theta/\phi$ [circle the correct variable(s)] in B whereas $\vec{p}$ is a vector function of $\rho/\theta/\phi$ in N.

(e) Alternately, the ***Cartesian coordinates*** x, y, z locate P from N_o as $\vec{p} = x\,\hat{n}_x + y\,\hat{n}_y + z\,\hat{n}_z$. Express x, y, and z in terms of ρ, θ, and ϕ.

Result: $x = \rho\sin(\theta)\sin(\phi) \qquad y = \qquad z = \rho\cos(\phi)$

(f) Express ρ, θ, ϕ in terms of x, y, z and calculate ρ, θ, ϕ when $x = 0$, $y = 0$, and $z = 1$. Note: The `atan2` function is described in Section 1.5.4 and is undefined if $x = y = 0$.

Result:
$$\rho = +\sqrt{\quad + \quad + z^2} \qquad \theta = \texttt{atan2}(x,\,y) \qquad \phi = \text{acos}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$
$$\rho = 1 \qquad \theta = ?? \qquad \phi = 0^\circ$$

(g) The motion of a particle P that is tied by a 2 m string to N_o can be described with the two angles θ and ϕ. Assuming $\hat{n}_z$ is ***vertically upward***, what location of P is likely to be encountered during the motion of P in N which would cause θ to be undefined?

Result: P will likely pass directly below N_o (or come to rest there), where θ is undefined. Also, P may pass above N_o where θ is undefined.

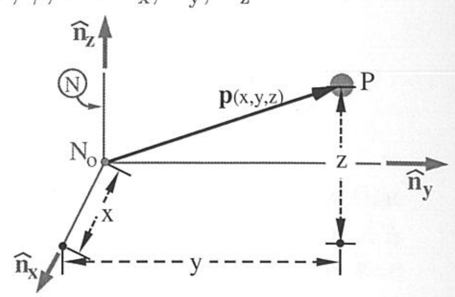
(h) Assuming $\hat{n}_z$ is ***horizontally right***, what locations of P cause θ to be undefined?[4]

Result: If P passes directly to the right or left of N_o, θ is undefined.

[4]The motion of P in N can be described without "singularities" by using the Cartesian Coordinates x, y, z and imposing the ***configuration constraint*** $x^2 + y^2 + z^2 = 4\text{ m}^2$. The idea of using additional coordinates to avoid singularities is central to understanding the use of ***four*** Euler parameters to describe the general ***3D*** orientation of a rigid body in space.

5.23 Spherical coordinates and vector differentiation via definition. (Section 6.3).

(a) Referring to Homework 5.22, use the definition of a vector derivative [equation (6.3)] to find the time-derivative in N of $\vec{\mathbf{p}}$ and express it in terms of ρ, θ, ϕ, $\dot\rho$, $\dot\theta$, $\dot\phi$, and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$.
Result:



$$\frac{{}^N d\vec{\mathbf{p}}}{dt} = \left[\dot\rho \sin(\theta)\sin(\phi) + \rho\cos(\theta)\sin(\phi)\,\dot\theta + \rho\sin(\theta)\cos(\phi)\,\dot\phi\right]\widehat{\mathbf{n}}_x$$

$$+ \qquad\qquad \widehat{\mathbf{n}}_y$$

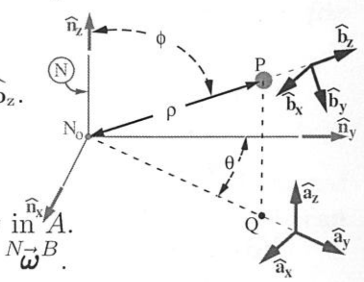
$$+ \qquad\qquad \widehat{\mathbf{n}}_z$$

(b) Calculate $\frac{{}^N d\vec{\mathbf{p}}}{dt}$ by using the ${}^nR^b$ rotation table to express $\frac{{}^N d\vec{\mathbf{p}}}{dt}$ in terms of $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ and then doing ***laborious*** trigonometric simplifications. (Attempt this until it is clear how laborious this is.)
Result:

$$\frac{{}^N d\vec{\mathbf{p}}}{dt} = \rho\sin(\phi)\,\dot\theta\,\widehat{\mathbf{b}}_x + \rho\dot\phi\,\widehat{\mathbf{b}}_y + \dot\rho\,\widehat{\mathbf{b}}_z$$

(c) The expression for $\frac{{}^N d\vec{\mathbf{p}}}{dt}$ is simpler when expressed in terms of $(\widehat{\mathbf{b}}_x, \widehat{\mathbf{b}}_y, \widehat{\mathbf{b}}_z)$ / $(\widehat{\mathbf{n}}_x, \widehat{\mathbf{n}}_y, \widehat{\mathbf{n}}_z)$.

5.24 Spherical coordinates and vector differentiation via angular velocity.

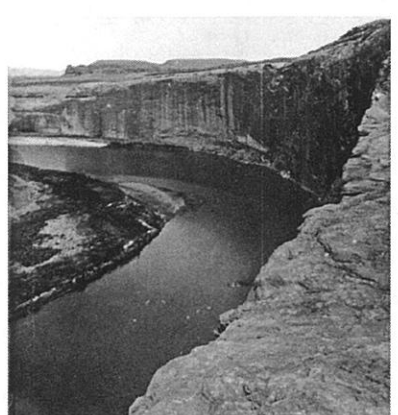


(a) **Inspect** the figure to determine P's position vector from N_o.
Calculate $\vec{\mathbf{p}}$'s time-derivative in B. Express results in terms of $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.
Results: $\vec{\mathbf{p}} = \qquad \widehat{\mathbf{b}}_z \qquad \frac{{}^B d\vec{\mathbf{p}}}{dt} =$

(b) Given below are A's angular velocity in N and B's angular velocity in A.
Complete the ***angular velocity addition theorem*** (below) to find ${}^N\vec{\boldsymbol{\omega}}^B$.
Result:
${}^N\vec{\boldsymbol{\omega}}^A = -\dot\theta\,\widehat{\mathbf{a}}_z \qquad {}^A\vec{\boldsymbol{\omega}}^B = -\dot\phi\,\widehat{\mathbf{b}}_x \qquad {}^N\vec{\boldsymbol{\omega}}^B = {}^N\vec{\boldsymbol{\omega}}^A + {}^A\vec{\boldsymbol{\omega}}^B = \qquad \widehat{\mathbf{a}}_z +$

(c) Use the ***golden rule for vector differentiation*** (shown below-left) to calculate the time-derivative of $\vec{\mathbf{p}}$ in N. Express results in terms of $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.
Result:
$$\frac{{}^N d\vec{\mathbf{p}}}{dt} = \frac{{}^B d\vec{\mathbf{p}}}{dt} + {}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{p}} \qquad \frac{{}^N d\vec{\mathbf{p}}}{dt} = \rho\sin(\phi)\,\dot\theta\,\widehat{\mathbf{b}}_x + \qquad \widehat{\mathbf{b}}_y + \qquad \widehat{\mathbf{b}}_z$$

(d) Relative to your work using the definition of vector differentiation in Homework 5.23b, the golden rule for vector differentiation is an **easier/harder** way to calculate $\frac{{}^N d\vec{\mathbf{p}}}{dt}$.



Courtesy USGS. Spherical coordinates help predict river flow and bank erosion on spherical Earth

· Ryanair ⇒ compagnie de vol (18€ strasbourg-Londres)
· Blablacar (37€ amsterdam-strasbourg)

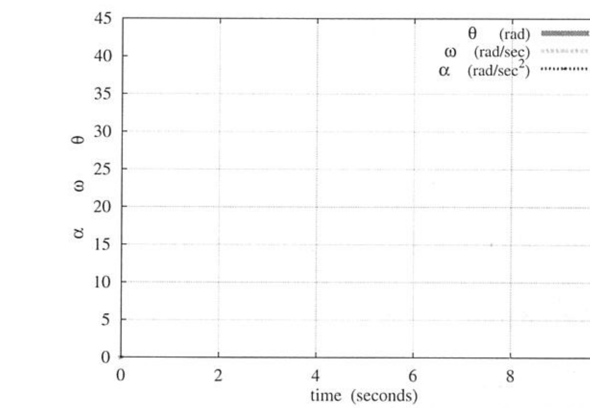
6.1 FE/EIT Review – Motion graph: $\boxed{\mathrm{T} \Rightarrow \alpha \Rightarrow \omega \Rightarrow \theta}$

The following wind turbine generates electricity from time-dependent aerodynamic wind forces. The wind creates a torque modeled as $\mathrm{T} = 20\ \frac{\mathrm{N\,m}}{\mathrm{sec}} * t$.

Measures of the wind turbine's angular acceleration α, angular velocity ω, and angle θ are governed by

$$\boxed{\mathrm{T} \underset{(2D)}{=} \mathrm{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(2D)}{=} \frac{d\theta}{dt}}$$

where $\mathrm{I} = 80\ \mathrm{kg\,m^2}$ is the turbine's relevant moment of inertia. Graph α in $\frac{\mathrm{rad}}{\mathrm{sec}^2}$, ω in $\frac{\mathrm{rad}}{\mathrm{sec}}$, and θ in rad for $0 \le t \le 8$ sec. Use initial values of $\omega = 0$ and $\theta = 0$.



6.2 Drawing a reference frame and unit vector bases. (Section 7.2).

<u>Draw</u> a reference frame or rigid body B shaped like a uniform-density doughnut (having a hole).
<u>Draw</u> a right-handed orthogonal bases fixed in B having unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.
<u>Draw</u> a different right-handed orthogonal bases fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.
<u>Draw</u> a properly located center of mass symbol ◕ and label this point as B_{cm}.
<u>Draw</u> a different point B_o fixed on B.

6.3 Notation, words, and pictures for rotation matrices, angular velocity, angular acceleration.

${}^bR^a$ – Description (words)	${}^N\vec{\boldsymbol{\omega}}^B$ – Description (words)	${}^N\vec{\boldsymbol{\alpha}}^B$ – Description (words)
${}^bR^a$ – draw b and a	${}^N\vec{\boldsymbol{\omega}}^B$ – draw B and N	${}^N\vec{\boldsymbol{\alpha}}^B$ – draw B and N

6.4 Definitions of angular velocity. (Section 7.3.6).
The definition of angular velocity of $\vec{\boldsymbol{\omega}} \triangleq \dot\theta\,\vec{\mathbf{k}}$ is a functional operational definition, i.e., in general, it is useful for calculating angular velocity and proving its properties (2D or 3D). **True/False**